Analysis of Some Adaptive, Feedback Based Schemes with Applications to High Speed Data Networks

G. I. Mousadis¹, T. A. Tsiligirides¹ and M. P. Bekakos²

¹Info Lab, Dept. of General Sciences, Agricultural University of Athens, Athens, Greece
²Dept. of Informatics, Athens University of Economics and Business, Athens, Greece

Abstract

Some new results are derived by modeling a class of rate-based feedback schemes in the support of data transport services in long-haul, high speed, data networks. In such an environment, traffic pattern varies often unpredictably, while the large propagation delay is becoming a major factor. The network configuration examined consists of three nodes and two paths, each path having a different propagation delay and many small VCs. Depending on the intensity of the cross traffic the entire network may be viewed as a collection of disjoint subnetworks each of which views the rest of the network as being represented by cross traffic streams. Based on the models already developed in [5] and the solution method proposed in [6] and [7], the reduced network model may be solved in a straightforward manner. The so obtained results are in full agreement with those already derived by simulation and are reported in the literature. As it appears, oscillations are controlled to acceptably small magnitude and therefore, such methods may be used as a powerful tool for optimum performance, by means of stabilizing the channel output and achieving fairness.

1. INTRODUCTION

On going research on feedback-based, flow control algorithms for the adaptive allocation bandwidths to VCs (Virtual Circuits), in high speed data networks and ATM (Asynchronous Transfer Mode) systems, has been stymied by the unavoidable limitation of large propagation delays. In particular, in the high speed environment, the time constants associated with the adaptive process are of the order of the propagation delays. Since there are some bursty applications where the bandwidth demand is for time periods which are comparable to the above time constants, the feedback flow control analysis appears to be an important supportive tool (rf. [1], [2]). However, the model analysis of the networks considered here, as well as of those considered in [6] and [7], is based on packet-by-packet adaptation and it is driven by response time measurements (rf. [3]-[5]). In all these network models acknowledgment packets carry information about the state of the queue at the bottleneck node and this information is used to control the transmission rate through a control function which may not be linear.

The model analyzed in this research is presented in Fig. 1 and consists of three nodes and two paths, each path having a different propagation delay and many small VCs per path. It is assumed that one of the paths has a long propagation delay \( \tau_L \) and it consists of \( C_L \) VCs, while the other one has a short propagation delay \( \tau_S \) and it consists of \( C_S \) VCs. Thus, the total number of VCs in the two paths is \( C = C_L + C_S \). Since the network, however, is assumed to be balanced many interactions may occur, but depending on the inten-
sity of the cross traffic, the number of bottleneck nodes on each path may be either one or two. Thus, the entire network may be viewed as a collection of disjoint subnetworks each of which views the rest of the network as being represented by cross traffic streams. As it may be observed, nodes 2 and 3 are lightly loaded and consequently the network may be reduced to a simpler configuration. The investigation of some interesting cases will follow, in which the number of VCs per path and the design objectives $T_L/T_S$ may vary; $T_i$ represents the throughput of the $i^{th}$ VC, $i=L, S$.

![Diagram](image)

The main contribution of this work is the extension of the method, already developed in 
[7], for more complex networks and its numerical testing. In Section 2, the application of the numerical method to the basic model is discussed; the case of a single node having one path with one VC traversing the single node is investigated. The method can be extended in a straightforward manner to cover the tandem model, as well as the single-hop with multiple VCs model, presented in Fig. 2. A complete analysis of the above configurations may be found in [6] and [7], where some numerical results are also reported and discussed. Further, in Section 3, the analysis is extended to the complex model depicted in Fig. 1. Also, since the network configuration considered is similar to the one considered in [4], the semi-analytical results obtained are compared against the simulation results presented therein. As it can be observed, the results achieved are in full agreement, consistency and accuracy with the simulation results for most of the examined cases. The paper concludes with some hints for further enhancement of the work presented herein.

2. THE MODEL

We consider the model presented in Fig. 2. This is a single-hop with multiple VCs network. The network is assumed to be balanced, namely, $\lambda = \mu - v$, where $v$ is the *exogenous Poisson cross traffic rate* and $\mu$ is the fixed *service rate*. For the $j^{th}$ VC, $j = 1, ..., C$, data experience some propagation delay $\tau_{0j}$, before returning to the source, with $\tau_j = \tau_{1j} + \tau_{0j}$, for each $j = 0, 1, ..., C$. Note that, the $\tau_j$'s are assumed to be $O(1)$ constants, so they do not scale with the large parameter $\lambda$ under the moderate framework in use.
Let $\theta_j > 0$, $j = 1, \ldots, C$, with $\sum_{j=1}^{C} \theta_j = 1$, denote a set of the desired fractional bandwidth allocation parameters to the VCs, in which $\theta_j$ is the fraction of the total throughput $T_j$, which is to be allocated to the $j$th VC. Then, for large $\lambda$ and all the pairs of $i$ and $j$ it holds:

$$\frac{T_i}{T_j} = \frac{\theta_i}{\theta_j} + O\left(\frac{1}{\sqrt{\lambda}}\right), \text{ or } T_j = \theta_j \lambda + O\left(\sqrt{\lambda}\right)$$

If the transmission rate, the service rate and the buffer occupancy at the $j$th VC are correspondingly denoted by, $\lambda_j(t)$, $\mu_j$ and $x_j(t)$, then the following system of DDEs (Delay Differential Equations) is obtained:

$$\frac{d}{dt} x_j(t) = \begin{cases} 
\lambda_j(t - \tau_{ij}) - \mu_j & \text{if } x_j(t) > 0, \text{ or } \lambda_j(t - \tau_{ij}) - \mu_j > 0 \\
0 & \text{otherwise}
\end{cases} \quad (1)
$$

$$\frac{d}{dt} \lambda_j(t) = \begin{cases} 
-\frac{\gamma_j}{\tau_j} \left[ x_j(t - \tau_{0j}) \sqrt{\lambda_j(t) - b_j} \right] \lambda_j(t - \tau_j), & \text{if } x_j(t - \tau_{0j}) > 0 \\
-\frac{\gamma_j}{\tau_j} \left[ x_j(t - \tau_{0j}) \sqrt{\lambda_j(t) - b_j} \right] \mu_j, & \text{otherwise}
\end{cases} \quad (2)
$$

In the above:

$$b_j = \sqrt{M_j \frac{\theta_j}{\sum_{i=1}^{C} \theta_i / \tau_i}}, \quad \forall \ j = 1, 2, \ldots, C, \quad (3)$$

where $M_j$ is the number of nodes in the $j$th VC (in this case, $M_j = 1$, $\forall j = 1, 2, \ldots, C$). Thus, the model in Fig. 2 can also cover the tandem network configuration. Note that, the system

Fig. 2: The single hop model: Multiple VCs with various propagation delays $\tau_j$ ($\tau_j = \tau_{ij} + \tau_{0j}$), $j = 1, 2, \ldots, C$ and individual windows.
may be further simplified by assuming that, \( \lambda_j = \mu_j + O(\sqrt{\mu_j}) \), \( \forall j=1,\ldots,C \). A detailed analysis including the stability conditions is given in [5].

In the analysis that follows and in order to avoid any further complexity, the basic model to be considered is consisting of a single bottleneck node with a single VC. This simple fluid model is governed by the following system of DDEs (the notation is simplified in a straightforward manner):

\[
\frac{d}{dt} x(t) = \begin{cases} 
\lambda(t - \tau_i) - \mu, & \text{if } x(t) > 0, \text{ or } \lambda(t - \tau_i) - \mu > 0 \\
0, & \text{otherwise}
\end{cases}
\]

\[
\frac{d}{dt} \lambda(t) = \begin{cases} 
-\frac{\gamma}{\tau} \left[ \frac{x(t - \tau_0)}{\mu \sqrt{\tau}} \sqrt{\lambda(t) - b} \right] \lambda(t - \tau), & \text{if } x(t - \tau_0) = 0 \\
-\frac{\gamma}{\tau} \left[ \frac{x(t - \tau_0)}{\mu \sqrt{\tau}} \sqrt{\lambda(t) - b} \right] \mu, & \text{otherwise},
\end{cases}
\]

which becomes linear by assuming that \( \lambda = \mu + O(\sqrt{\mu}) \). The resulting system of DDEs has been solved in [6] and it has been further applied in [7], where some numerical results have also been reported. Next, the main steps for solving the above system of DDEs are reviewed.

Using the transformation \( x_\epsilon(t) = x(t - \tau_2) \), \( \forall t \in [\tau_0, \infty) \), the system may be re-written as it follows:

\[
\frac{d}{dt} x_\epsilon(t) = \begin{cases} 
\lambda(t - \tau_i) - \mu, & \text{if } x_\epsilon(t) > 0, \text{ or } \lambda(t - \tau_i) - \mu > 0 \\
0, & \text{otherwise}
\end{cases}
\]

\[
\frac{d}{dt} \lambda(t) = \begin{cases} 
-\frac{\gamma}{\tau} \left[ \frac{x_\epsilon(t)}{\mu \sqrt{\tau}} \sqrt{\lambda(t) - b} \right] \lambda(t - \tau), & \text{if } x_\epsilon(t) = 0 \\
-\frac{\gamma}{\tau} \left[ \frac{x_\epsilon(t)}{\mu \sqrt{\tau}} \sqrt{\lambda(t) - b} \right] \mu, & \text{otherwise}.
\end{cases}
\]

Thus, in case that \( x_\epsilon(t)=0 \), the exact value of \( \lambda(t) \) is obtained through the use of the Laplace transforms as it follows:

\[
\lambda(t) = c \sum_{n=0}^{[\tau/\tau]+1} \frac{\rho^n (t - (n - 1)\tau)^n}{n!},
\]

where \([ \ ] \) denotes the integer part and \( \lambda(0)=c \), with \( c \) constant in \([-\tau, 0] \). However, in case that \( x_\epsilon(t)>0 \), the assumption \( \lambda = \mu + O(\sqrt{\mu}) \) may be used, in order to calculate \( \lambda(t) \) through the Euler form. This scheme is called the MTB-1 and it is given by:

\[
\lambda(t + h) = \lambda(t) + h \frac{\gamma \mu}{\tau} \left( \frac{x_\epsilon(t)}{\sqrt{\mu \tau}} - 1 \right) + O(\sqrt{\mu}),
\]
where $h$ is the step size. Note that, a further improvement may be achieved in case that the extended Euler method (i.e., predictor corrector method) is used, which is defined as it follows:

$$
\lambda(t + h) = \lambda(t) + \frac{h}{2} \left( \frac{d}{dt} \lambda(t) + \frac{d}{dt} \lambda(t + h) \right)
$$

Therefore,

$$
\frac{d}{dt} \lambda(t + h) = -\frac{\gamma \mu}{\tau} \left( \frac{x_e(t + h)}{\mu \sqrt{\tau}} \sqrt{y(t + h)} - \frac{\lambda(t) - \mu}{\tau} \right),
$$

with

$$
x_e(t + h) = x_e(t) + h(\lambda(t - h) - \mu) \quad \text{and} \quad y(t + h) = \lambda(t) - h \frac{\gamma \mu}{\tau} \left( \frac{x_e(t)}{\mu \sqrt{\tau}} \sqrt{\lambda(t)} - 1 \right)
$$

3. NUMERICAL RESULTS

The model under investigation consisted of three nodes and two paths, namely, the Short path with $\tau_S = 0.5\tau$ and the Long path with $\tau_L = \tau$. The design objectives assumed either fair allocation of the bandwidth, i.e., $T_L/T_S = 1$, or arbitrary, i.e. $T_L/T_S = 1.5$ or 0.67. Many cases were examined using various numbers of smaller VCs and cross traffic rates. In all the cases, it was assumed that the data rates were of 45 Mbps and $\tau = 47$ms. Thus, with a packet size of 1000 bytes, $\mu \approx 264$Kbytes/$\tau$. The results are tabulated as it follows.
In Table 1, the cases 1 to 3 correspond to the Group V-Table 5 of [4] and present the results obtained when there is no cross traffic and there is only a single VC on each of the two paths, namely, \((C_L, C_S) = (1, 1)\). The different cases considered were due to the different objectives for the values of \(T_L/TS\), namely, 1.0, 1.5 and 0.67. The design parameters \(b_L\) and \(b_S\) have been calculated with formula (3) using the appropriate bandwidth allocation parameters and taking into account that the nodes 2 and 3 were lightly loaded, namely, \(M_j=1, j=S, L\). Thus, the network was reduced to the model of Fig. 3(a). It is interesting to note that, the calculated throughput values \(T_L/TS\), obtained using methods MTB-1 (MTB1 column) and MTB-2 (MTB2 column), were very close to the design objectives, denoting the correct choice of the design parameters. This was also verified from the results obtained in [4] (MS column).

Further, in Table 1, the cases 4 to 6 correspond to the Group VI-Table 6 of [4] and present the results obtained when there is no cross traffic and there are multiple small VCs on each path, namely, \((C_L, C_S) = (8, 16)\). The procedure to calculate the design parameters \(b_L\) and \(b_S\) was left unaltered, namely, nodes 2 and 3 were lightly loaded. The calculated throughput values \(T_L/TS\), realized by the methods MTB-1 and MTB-2, were very close to the design objectives, while a very good agreement between the results obtained, particularly in the case of the MTB-2 method, with those obtained through simulation was observed.

In Table 2, the cases 1 to 3 correspond to the Group VII-Table 7 (cases 4, 5 and 6) of [4] and present the results obtained using only one VC on each path and various cross traffic rates. The procedure to obtain the design parameters \(b_L\) and \(b_S\) was the same, but due to the cross traffic rate different values for \(M_j\) were applied. For example, if both nodes 2 and 3 were lightly loaded, then these nodes were ignored; thus, \(M_L = 1, M_S = 1\) and the equivalent network configuration is presented in Fig. 3(a). Similarly, if only one of the nodes 2, 3 were lightly loaded, this particular node was ignored; therefore, \(M_L = 1, M_S = 2\) or \(M_L = 2, M_S = 1\) and the equivalent network configuration is presented either in Fig. 3(b) or 3(c), respectively. A last case, not included in the simulated experimentation of [4], was that neither of the switches 2, 3 was lightly loaded and thus \(M_L=2, M_S=2\), which resulted in the network configuration of Fig. 3(d). In all the cases considered, the calculated throughput values \(T_L/TS\), realized by method MTB-2 (MTB-1 was excluded from the analysis), were very close to 1, which was the design objective.

Finally, in Table 2, cases 5, 6 and 7 correspond to the Group VIII-Table 8 (cases 4, 5 and 6) of [4] and present the results obtained using multiple VCs on each path, namely, \((C_L, C_S) = (8, 16)\) using various cross traffic rates. The design parameters \(b_L\) and \(b_S\) were obtained using formula (3), but due to the different set of cross traffic intensities applied to each node, and due to the differences in the bandwidth allocation parameters, different values for \(M_j, j = S, L\) were obtained. In Table 2 (cases 4, 8, 9 and 10) some further results are also given, which were not included in the simulated experimentation of [4], but seemed to be of practical interest. However, in all the cases considered, the throughput values \(T_L/TS\), realized by method MTB-2, were very close to 1, which was the design objective.
4. CONCLUSIVE REMARKS

Herein, the adaptive algorithm developed in [7] was applied to some complex, delayed, feedback schemes and very interesting experimental results were achieved. A major problem in such high speed VC data networks is the statistical interaction among the traffic flows to be controlled and the large propagation delay. The proposed algorithm was tested using some well known network configuration schemes (rf. [4]) and the numerical results achieved were in full agreement with the reported simulation results. These results also indicated the appropriate scaling of the parameters so as to make such schemes to perform near their optimal values, stabilizing the channel output and achieving fairness.

To conclude, it appears that the MTB semi-analytical algorithm may also be used with the connection admission control procedure, to predict the maximum and minimum source transmission rates. This will assist in reserving some network resources, before negotiating with a new network user for a specific service scheme. In a similar way, applying the MTB-2 scheme to the single-hop model, an Explicit Rate switch algorithm could be proposed to support the ABR (Available Bit Rate) service of the ATM networks. This scheme is currently under investigation.

REFERENCES

### Table 1: Calculation of the parameters

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### Table 2: Node 2

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### Table 3: Node 3

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**Table 2:** Experimental results for the three nodes and 2 VCs (long and short path) model. The number of small VCs is (C_L, C_S)=(1, 1): cases 1, 2, 3 and 4, and (C_L, C_S)=(8, 16): cases 6, 7, 8, 9 and 10.

Design objectives: T_L/T_S=1.