Analysis of a *p*-persistent CSMA packetized cellular network with capture phenomena

T Tsiligirides and D G Smith* study the capture effect on a *p*-persistent CSMA protocol for a packetized mobile cellular network

A p-persistent CSMA protocol suitable for packetized mobile radio-telephone cellular networks with capture and a number of mobile terminals transmitting to a common receiver is considered. In a micro-cellular environment, messages from different transmitting terminals will suffer different attenuations (path loss and fading phenomena), yielding different levels of energy at the receiver. Thus, one out of 'i' attempting transmission packets will be successfully transmitted with some capture probability f_i. Assuming that the receiver is within the range and in line-of-sight of all the mobile users in the cell, the capture probabilities can be determined by means of simulation, and a new probabilistic Markov model for the above network is introduced and analysed using the slot property of the channel. Under moderate packet length size the results obtained in respect of the throughput-delay performance and stability under heavy traffic conditions are promising compared with those of the corresponding conventional p-persistent CSMA schemes.

Keywords: Markov chain analysis, packetized mobile network, capture, CSMA protocol, simulation, performance analysis

Informatics Laboratory, Agricultural University of Athens, Iera Odos 75, 118 55 Athens, Greece
*Department of Electrical and Electronic Engineering, University of Strathclyde, 204 George Street, Glasgow G1 1XW, UK
Paper received; 25 May 1990. Revised paper received: 27 October 1990

MODEL DESCRIPTION

Consider a finite population of N mobile users sharing one channel for transmission to one receiver (base station). The channel is slotted with slot duration equal to T seconds. Any effect due to the propagation delay is assumed negligible. Each user generates packets of fixed length equal to $\theta \geqslant 1$ slots. Thus, the packet transmission time if θT seconds.

Each user stores and attempts to transmit at most one packet for each time slot. A user without a packet in his buffer is said to be free; if he has a packet he is said to be ready. We assume a synchronized structure. Each user generates (or does not generate) a packet in a time slot, independently of other users, with probability σ (or $1 - \sigma$). Packet arrivals occur only from free users at the kT instants $;k = 1,2,\ldots,$ and their number is distributed according to the binomial distribution $b(N,j,\sigma) = \binom{N}{j}\sigma^j (1-\sigma)^{N-j}$ $;j = 0,1,2,\ldots,N$. Packets specified above and generated from ready users are assumed lost.

A collision does not necessarily result in the destruction of all the packets involved[1-5] The probability that exactly one out of i attempting transmission packets will be successfully transmitted is denoted by f_i $;i = 0,1,2,\ldots,N$ with $f_0 = 0$, $f_1 = 1$, and $f_i \varepsilon (0,1)$ $;i = 2,3,\ldots,N$. If a collision results in no successful transmission the event is referred to as a collision with no successful transmission, otherwise, if a collision results in a successful transmission the event is referred to as a collision with a successful transmission.

It is assumed that the base station is within the range and line-of-sight of all the mobile users in the cell. All the mobile users are aware of the status of the base station (busy or idle) in each time slot through a separate, and independent, secondary channel, and behave accordingly. Thus, if a collision with no successful transmission occurs, the channel is seen to be free immediately, and all the ready mobile users will compete again in the following time slot. Alternatively, if a collision results in a successful transmission, the transmission period of θ time slots starts, during which time no attempts are made by the mobiles.

Any ready mobile receives the information about the status of the channel through the secondary channel in each time slot and proceeds as follows. If the status is idle the contention period starts, during which all the ready mobiles attempt to transmit their packets with probability δ. Alternatively, if the status is busy, this is due to a successful transmission and all the ready mobiles postpone their attempts up to the time slot at which the transmission is finished.

Note that in the last time slot of the transmission period the base station changes its status from busy to idle, and informs all the ready mobiles, through the secondary channel, accordingly. Thus, attempts with probability δ by the ready mobiles will be allowed from the next time slot, when the transmission is due to finish and the contention period begins. Inevitably, during this period collisions with no successful transmission will occur, but due to the different levels of energy between the attempting transmission packets at the base station, one of the colliding packets will be successfully transmitted in some time slot. This event terminates the contention period and the transmission period will start in this time instant.

MODEL ANALYSIS

In Figure 1 the operation of the system is presented[6]. The slotted time axis is divided into cycles, each one of which consists of three periods: the idle, the contention and the transmission periods. We proceed by giving the exact definitions.

A cycle is defined as the elapsed time (in slots) between the time instants in which two consecutive successfully transmitted packets and their transmission. The idle period of a cycle is defined as the time interval (in slots) between the time instant when the successfully transmitted packet of the previous cycle ends its transmission leaving behind no ready users, and the time instant when one or more new packets arrive. If, by the end of the transmission period of the previous cycle, one or more ready users are left behind, the idle period is zero and the system immediately enters into the contention period.

Depending on whether the idle period of a cycle is zero or not, there are two cases in which the contention period can be defined. Thus, if the idle period is zero, the contention period is defined as the time interval (in slots) between the time instant when the successfully transmitted packet of the previous cycle ends its transmission leaving behind one or more ready users (beginning of a new cycle), and the time instant when the first collision with a successful transmission occurs. Alternatively, if the idle period is not zero, the contention period is defined as the time interval (in slots) between the time instant when the idle period ends leaving behind one or more ready users, and the time instant when the first collision with a successful transmission occurs. The contention period is zero if a collision with a successful transmission occurs at the first time instant. Finally, the transmission period of a cycle immediately follows the contention period and has fixed duration of θ slots.

It is now evident that the state of the system can be described through the number of ready users left behind by the end point of each cycle. To derive the transition matrix of the system one has to find out the number of ready users left behind by the end point of each period and subsequently in each time slot. Then the transition

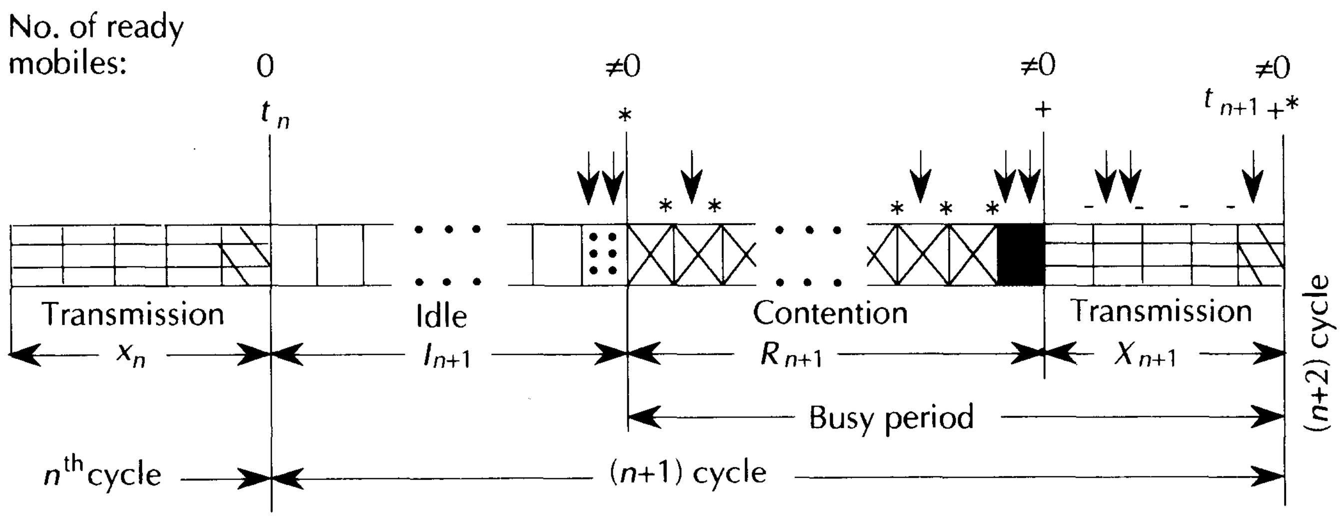

Figure 1. Channel operation. ∗(+): collision with no (a) successful transmission occured; (-): no attempt is allowed by any mobile user

matrix will be calculated as a product of single slot transition matrices. In the following the transition matrices of each period are derived.

Transition matrix of the cycle's idle period

Let us assume that the idle period of a cycle has k single slots duration. In each one of the first $k-1$ slots no packet arrival occurs, and therefore the total number of ready users in the system remains at zero. In the last time slot some fresh packet arrivals occur, and therefore this number changes to some non-zero value. Let $\mathbf{H}_- = ({}^-h_{ij})$ be the one step transition matrix describing the state of the system in each one of the first $k-1$ single slots, and $\mathbf{H}_+ = ({}^+h_{ij})$ be the one step transition matrix for the last single slot of the idle period. Then:

$$ {}^-h_{ij} = \begin{cases} b(N,0,\sigma) & \text{if } i = j = 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} $$

$$ {}^+h_{ij} = \begin{cases} b(N,j,\sigma) & \text{if } i = 0 \;; j = 1,2,\ldots,N \\ 1 & \text{if } i = j \;; i,j = 1,2,\ldots,N \\ 0 & \text{otherwise} \end{cases} $$

All the Eigenvalues of the Matrix $\mathbf{H}_-$ are absolutely less than one, and therefore the series $\sum_{k=0}^{\infty} \mathbf{H}_-^k$ converges. Using the single slot property of the channel the transition matrix $\mathbf{H} = (h_{ij})$ of the idle cycle period is given by:

$$ \mathbf{H} = \sum_{k=1}^{\infty} \mathbf{H}_-^{k-1}\mathbf{H}_+ = (\mathbf{I} - \mathbf{H}_-)^{-1}\mathbf{H}_+ \text{ with elements} $$

$$ h_{ij} = \begin{cases} b(N,j,\sigma)/\{1 - b(N,0,\sigma)\} & \text{if } i = 0 \;; j = 1,2,\ldots,N \\ 1 & \text{if } i = j \;; i,j = 1,2,\ldots,N \\ 0 & \text{otherwise} \end{cases} $$

Note that $\mathbf{H1} = \mathbf{1}$ with $\mathbf{1}$ be a $(N+1) \times 1$ dimensions column vector of one's. All the involved matrices $\mathbf{H}$, $\mathbf{H}_-$, $\mathbf{H}_+$ and $\mathbf{I}$ are of $(N+1) \times (N+1)$ dimensions.

Transition matrix of the cycle's contention period

In a similar manner, let the contention period of a cycle have m single slots duration. Attempts are allowed in each one of the m time slots with probability δ by any ready user. In the first $m-1$ time slot all the attempts result in a collision with no successful transmission, while in the last slot a collision with a successful transmission occurs. Let $\mathbf{R}_- = ({}^-r_{ij})$ be the one step transition matrix describing the state of the system in each one of the first $m-1$ single slots, and $\mathbf{R}_+ = ({}^-r_{ij})$ be the one step transition matrix for the last single slot of the contention period, respectively. Then in case $\delta = 1$ (1-persistent CSMA protocol):

$$ {}^-r_{ij} = \begin{cases} b(N-i, j-i, \sigma)\{1 - f_j\} & \text{if } i = 2,3,\ldots,N \\ & \;; j = i, i+1, \ldots, N \\ 0 & \text{otherwise} \end{cases} $$

and:

$$ {}^+r_{ij} = \begin{cases} b(N-i, j-i+1, \sigma) f_{j+1} & \text{if } i = 2,3,\ldots,N \\ & \;; j = i-1, i, i+1, \ldots, N-1 \\ 1 & \text{if } i = j \;: i,j = 0,1 \\ 0 & \text{otherwise} \end{cases} $$

Alternatively, in case $\delta \neq 1$ (δ-persistent CSMA protocol), let $\mathbf{V}$ be the number of attempts made by the ready users in a time slot. Then:

$$ {}^-r_{ij} = \begin{cases} b(N-i, j-i, \sigma) \sum_{\mathbf{V}=0}^{j} b(j, \mathbf{V}, \delta)\{1 - f_j\} & \text{if } i = 1,2,\ldots,N \\ & \;; j = i, i+1, \ldots, N \\ 0 & \text{otherwise} \end{cases} $$

and:

$$ {}^+r_{ij} = \begin{cases} b(N-i, j-i+1, \sigma) \sum_{\mathbf{V}=0}^{j+1} b(j+1, \mathbf{V}, \delta) f_j & \text{if } ; i = 1,2,\ldots,N \\ & \;; j = i-1, i, i+1, \ldots, N-1 \\ 1 & \text{if } i = j = 0 \\ 0 & \text{otherwise} \end{cases} $$

Note that in both cases, the event of ending the contention period with N ready users left behind and having a collision with a successful transmission is impossible. Again, all the Eigenvalues of the matrix $\mathbf{R}_-$ are absolutely less than one, and thus the series $\sum_{m=0}^{\infty} \mathbf{R}_-^m$ converges. Using the single slot property of the channel the transition matrix of the contention period of a cycle $\mathbf{R} = (r_{ij})$ is calculated as follows:

$$ \mathbf{R} = \sum_{m=0}^{\infty} \mathbf{R}_-^{m-1}\mathbf{R}_+ = (\mathbf{I} - \mathbf{R}_-)^{-1}\mathbf{R}_+ \text{ with } \mathbf{R1} = \mathbf{1} $$

All the involved matrices $\mathbf{R}_-$, $\mathbf{R}_+$ and $\mathbf{R}$ are of $(N+1) \times (N+1)$ dimensions. Generally, the matrix $\mathbf{R}$ cannot be calculated explicitly.

Transition matrix of the cycle's transmission period

The transmission period of a cycle always has a fixed duration of θ time slots. During this period new arrivals from free users may occur, but attempts are not allowed. Let $\mathbf{X}_- = ({}^-x_{ij})$ be the one step transition matrix for each of the first $\theta - 1$ single slots and $\mathbf{X}_+ = ({}^+x_{ij})$ be the one step transition matrix for the last single slot of the transmission period, respectively. Then:

$$ {}^-x_{ij} = \begin{cases} b(N-i, j-i, \sigma) & \text{if } i = 1,2,\ldots,N \;; j = i, i+1, \ldots, N \\ 1 & \text{if } i = j = 0 \\ 0 & \text{otherwise} \end{cases} $$

and

$$ {}^-x_{ij} = \begin{cases} \sigma b(N-i, j-i, \sigma) + (1-\sigma) b(N-i, j-i+1, \sigma) & \text{if } ; i = 1,2,\ldots,N \\ & \;; j = i-1, i, i+1, \ldots, N \\ 1 & \text{if } i = j = 0 \\ 0 & \text{otherwise} \end{cases} $$

Note that in the last time slot of the transmission period an additional arrival, coming from the transmitting user, may or may not occur. Therefore, the number of ready users may or may not decrease by one.

Since the transmission period of a cycle is fixed, the transition matrix $\mathbf{X} = (x_{ij})$ is calculated as $\mathbf{X} = \mathbf{X}_-^{\theta-1}\mathbf{X}_+$.

System transition matrix

The chain defined between the consecutive end cycle points is Markovian. The system transition probability matrix $\mathbf{P} = (p_{ij})$ is calculated as a product of several single slot transition matrices:

$$\mathbf{P} = \mathbf{HRX} = (\mathbf{I} - \mathbf{H}_-)^{-1}\mathbf{H}_+(\mathbf{I} - \mathbf{R}_-)^{-1}\mathbf{R}_+\mathbf{X}_-^{\theta-1}\mathbf{X}_+$$

Therefore, the steady state distribution vector $\pi = (\pi_0, \pi_1, \ldots, \pi_N)$ can be obtained by solving the linear system $\left\{\pi = \pi\mathbf{P} \text{ with } \sum_{i=0}^{N} \pi_i = 1\right\}$.

Capture probabilities

There are several methods to calculate the capture probabilities $f_i; i = 0,1,2,\ldots,N$. The most efficient method requires explicit knowledge of the area covered by the radio-telephone network. Using topographical databases it is possible to create maps showing the mean levels of energy, produced at the base station (receiver) by a mobile user transmitting from different regions of the covered area. Information from statistical surveys is also used to estimate the effect of the different attenuation such as path loss and shadow or Rayleigh fading. The above information is combined with the relative number of mobile users expected in the respective regions to give the probability distribution $F(x) = Pr[E < x]$, where E is a random variable showing the level of power caused by a mobile.

Namislo[5] indicated that even in the most simple cases it is extremely difficult to calculate the exact values of the capture probabilities $f_i; i = 0,1,2,\ldots,N$. Alternatively, to calculate them by means of simulation he assumed that the users are spread uniformly over the whole region, and that their distances from the receiver are random variables with a common distribution function given by $G(x) = x^2$ $;x\varepsilon[0,1]$. If the level of power E_j caused by a transmitting mobile M_j at the receiver due to path loss is proportional to $1/x^m$, the base station will be able to capture this *transmission if* $E_j / \sum_{i=1}^{N} E_i \geqslant c; i \neq j$, for some constant $c \geqslant 1$ and the probabilities f_i can be determined using Monte Carlo simulation. Note that the exponent m is between 2 and about 5, depending on the geography of the region.

The simulation experiment assumes that k idendical ready mobile users are uniformly spread in a circular area of radius 1, which approximates the cell, with the base station located at the centre of the circle. It is also assumed that shadow or Reyleigh fading effects are neglected ($m = 2$). Then:

- Set $f_0 = 0.0$ and $f_1 = 1.0$.
- For each value of $k = 2,3,\ldots,N$ representing the number of ready mobile users M_k in the cell.
- Compute the Cartesian distances d_i and the level of power $E_i = 1/d_i^2; i = 1,2,\ldots,k$ of all the mobiles M_i from the base station.
- Check if $\max_i\{E_i\} \geqslant c \sum_{j=1}^{N} E_j; j \neq i$ (e.g. with $c = 2.0$ the corresponding signal-to-noise ratio is 3dB).
- Repeat the algorithm.

Then, the capture probabilities f_i are given as the relative number of times the above condition is satisfied.

PERFORMANCE MEASURES

In the following some measures of prime interest are calculated. These measures are the throughput, the mean busy and mean idle times of the channel, the mean backlog, the delay and the rejection probability. As is shown, the deriviation of these measures is based on the mean length of each one period and on the mean cycle length. The analysis further requires the conditional mean length of the idle and contention periods given the number of ready users at the beginning of the corresponding period. No analysis is needed for the transmission period.

Idle period mean length

To calculate the mean idle period length $\bar{C}idl$, let $\mathbf{C_I}$ be a $(N + 1) \times 1$ column vector with elements of the conditional mean idle period lengths, given that the system had i ($i = 0,1,2,\ldots,N$) ready users at the beginning of the idle period. Obviously, the only non-zero element of this vector corresponds to the state $i = 0$. Since $[\mathbf{H}_-^{k-1}\mathbf{H}_+\mathbf{1}]_{ij}$ is the conditional probability that the idle period length is k slots, given there were i packets at the beginning of this period and j packets at its end, the i^{th} element $[\mathbf{C_I}]_i$ of the column vector $\mathbf{C_I}$ is obtained as:

$$\begin{aligned}[\mathbf{C_I}]_i &= \sum_{k=1}^{\infty} k \sum_{j=0}^{N} [\mathbf{H}_-^{k-1}\mathbf{H}_+\mathbf{1}]_{ij} = \sum_{k=1}^{N} [k\mathbf{H}_-^{k-1}\mathbf{H}_+\mathbf{1}]_i \\ &= [\{(\mathbf{I} - \mathbf{H}_-)^{-1}\}^2\mathbf{H}_+\mathbf{1}]_i = [(\mathbf{I} - \mathbf{H}_-)^{-1}\mathbf{H1}]_i \\ &= [(\mathbf{I} - \mathbf{H}_-)^{-1}\mathbf{1}]_i\end{aligned}$$

where $\mathbf{1}$ is a column vector with $N + 1$ elements, all equal to 1. Note that since $|\mathbf{E}(\mathbf{H}_-)| < +\infty$, the matrix $(\mathbf{I} - \mathbf{H}_-)^{-1}$ always exists. Thus, the mean idle period length is given by $\bar{C}idl = \pi\mathbf{C_I} = \pi(\mathbf{I} - \mathbf{H}_-)^{-1}\mathbf{1}$.

Contention period mean length

In a similar manner let $\bar{C}con$ be the mean length of the contention period and $\mathbf{C_R}$ be a $(N + 1) \times 1$ column vector with elements of the conditional mean contention period lengths, given that the system had i ($i = 0,1,2,\ldots,N$) ready users at the beginning of the contention period. Obviously, the element corresponding to the state $i = 0$ is zero. Since $[\mathbf{R}_-^{m-1}\mathbf{R}_+\mathbf{1}]_{ij}$ is the conditional probability that the contention period length is m single time slots given there were i packets at the beginning of the period and j packets at its end, the i^{th} element $[\mathbf{C_R}]_i$ of the column vector $\mathbf{C_R}$ is obtained as:

$$[\mathbf{C_R}]_i = \sum_{m=1}^{\infty} k \sum_{j=0}^{N} [\mathbf{R}_-^{m-1}\mathbf{R}_+\mathbf{1}]_{ij}$$

$$= \sum_{m=1}^{N} [m\mathbf{R}_-^{m-1}\mathbf{R}_+\mathbf{1}]_i$$

$$= [\{(\mathbf{I} - \mathbf{R}_-)^{-1}\}^2\mathbf{R}_+\mathbf{1}]_i = [(\mathbf{I} - \mathbf{R}_-)^{-1}\mathbf{R1}]_i$$

$$= [(\mathbf{I} - \mathbf{R}_-)^{-1}\mathbf{1}]_i$$

Note that since $|E(\mathbf{R}_-)| < +\infty$, the matrix $(\mathbf{I} - \mathbf{R}_-)^{-1}$ always exists. Therefore, the mean contention period length is given by $\bar{C}con = \pi\mathbf{C_R} = \pi(\mathbf{I} - \mathbf{R}_-)^{-1}\mathbf{1}$.

Mean cycle length

The mean cycle length $\bar{C}$ consists of the mean idle period length $\bar{C}idl$, the mean contention period length $\bar{C}con$ and the fixed transmission period of θ time slots. If $\mathbf{C}$ is a $(N + 1) \times 1$ column vector such that $\mathbf{C} = \mathbf{C_I} + \mathbf{C_R} + \theta\mathbf{1}$, then:

$$\bar{C} = \pi\mathbf{C} = \pi\{\mathbf{C_I} + \mathbf{C_R} + \theta\mathbf{1}\} = \pi\mathbf{C_I} + \pi\mathbf{C_R} + \theta\pi\mathbf{1}$$

$$= \pi\{(\mathbf{I} - \mathbf{H}_-)^{-1} + (\mathbf{I} - \mathbf{R}_-)^{-1} + \theta\}1$$

Throughput

Each cycle carries a successful transmission of θ time slots. Therefore, the throughput of the system is defined as the ratio between the packet transmission time and the mean cycle length, $S_1 = \theta/\bar{C}$. Alternatively, the throughput can be defined as $S_2 = \theta/\{\bar{C} - \bar{C}idl\}$, namely the ratio between the packet transmission time and the mean length of the cycle the channel is actually busy (mean busy period length).

Mean busy and mean idle time

The mean busy time of the system, $Ubus_1$ say, is defined as the ratio between the mean busy (contention and transmission) period length and the mean cycle length. It consists of two parts the first of which, $Ucon_1$ say, is the proportion of the time spent in the contention period, while the second part is the throughput S_1 of the system. Therefore:

$$Ubus = Ucon_1 + S_1 = \bar{C}con/\bar{C} + \theta/\bar{C}$$

$$= \{\pi(\mathbf{I} - \mathbf{R}_-)^{-1}\mathbf{1} + \theta\}/\pi\{(\mathbf{I} - \mathbf{H}_-)^{-1}$$

$$+ (\mathbf{I} - \mathbf{R}_-)^{-1} + \theta\}1$$

Alternatively, the proportion of the time spent in the contention period can be defined by excluding the mean idle period length. In this case it will be:

$$Ucon_2 = \bar{C}con/(\bar{C} - \bar{C}idl)$$

$$= \pi(\mathbf{I} - \mathbf{R}_-)^{-1}\mathbf{1}/\pi\{(\mathbf{I} - \mathbf{R}_-)^{-1} + \theta\}\mathbf{1}$$

and:

$$Ucon_2 + S_2 = 1.$$

Finally, the mean idle time of the system, $Uidl$ say, is obtained as:

$$Uidl = 1 - Ubus = \bar{C}idl/\bar{C}$$

$$= \pi(\mathbf{I} - \mathbf{H}_-)^{-1}\mathbf{1}/\pi\{(\mathbf{I} - \mathbf{H}_-)^{-1}$$

$$+ (\mathbf{I} - \mathbf{R}_-)^{-1} + \theta\}1$$

Rejection probability

The rejection probability $Prej$ is obtained as the percentage of users that are not entered into the system. Therefore, $Prej = (N\sigma - 1/\bar{C})/N\sigma = 1 - 1/\sigma\bar{C}$.

Mean backlog

The mean backlog is defined as the mean number of ready users in the system, and it corresponds to the mean queue length, or equivalently, to the mean number of packets in the system. It is computed either as the ratio between the expected number of ready users, $\bar{B}bus$ say, over all slots in the busy period of a cycle, and the mean cycle length $\bar{C}$, or as the ratio between the $\bar{B}bus$ again and the mean busy period length $\bar{C}bus = \bar{C} - \bar{C}idl$. Therefore:

$$\bar{N}_1 = \bar{B}bus/\bar{C} \text{ or } \bar{N}_2 = \bar{B}bus/\bar{C}bus \text{ with}$$

$$\bar{B}bus = \bar{B}con + \bar{B}tra = \pi\mathbf{B_R} + \pi\mathbf{B_X}$$

Note that $\bar{B}con = \pi\mathbf{B_R}$ and $\bar{B}tra = \pi\mathbf{B_X}$ are the expected number of ready users in the contention and transmission periods, respectively. Moreover, $\mathbf{B_R} = ([\mathbf{B_R}]_i)$ and $\mathbf{B_X} = ([\mathbf{B_X}]_i)$;$i = 0,1,2,\ldots,N$ are $(N + 1) \times 1$ column vectors with elements the average number of ready users over all slots in the contention or transmission period, respectively, given that the system had i ready users at its beginning. Thus, if $\mathbf{j} = (0,1,2,\ldots,N)^T$ is a $(N + 1) \times 1$ column vector with elements the number of ready users corresponding to the state i then:

$$[\mathbf{B_R}]_i = \sum_{q=0}^{N} [\mathbf{H}]_{iq} \sum_{m=1}^{\infty} \sum_{j=0}^{N} j[\mathbf{R}_-^{m-1}]_{qj}$$

$$= [\mathbf{H} \sum_{m=1}^{\infty} \mathbf{R}_-^{m-1} j]_i$$

$$[\mathbf{B_X}]_i = \sum_{q=0}^{N} [\mathbf{H}]_{iq} \sum_{r=0}^{N} [\mathbf{R}]_{qr} \sum_{n=1}^{\theta} \sum_{j=0}^{N} j[\mathbf{X}_-^{n-1}]_{rj}$$

$$= [\mathbf{HR} \sum_{n=1}^{\theta} \mathbf{X}_-^{n-1} j]_i$$

Note that the number of ready users during the idle period is zero. The expected number of ready users during the contention and the transmission periods is given, respectively, as:

$$\bar{B}con = \pi \mathbf{B_R} = \pi \mathbf{H}(\mathbf{I} - \mathbf{R}_-)^{-1}\mathbf{j} \text{ and}$$

$$\bar{B}tra = \pi \mathbf{B_X} = \pi \mathbf{HR} \sum_{n=1}^{\theta} \mathbf{X}_-^{n-1}\mathbf{j}$$

while the mean backlog in the contention, $\bar{N}con$ say, and the transmission, $\bar{N}tra$ say, periods can be obtained, respectively, as:

$$\bar{N}con = \bar{B}con/\bar{C}bus \text{ and } \bar{N}tra = \bar{B}tra/\bar{C}bus \text{ with}$$

$$\bar{N}_2 = \bar{N}con + \bar{N}tra$$

Mean packet delay

The mean packet delay, $\bar{D}$ say, can be derived through Little's formula. Since $\bar{N}_1/S_1 = \bar{N}_2/S_2 = \bar{B}bus/\theta$ the mean delay is given by:

$$\bar{D} = \bar{B}bus/\theta = \pi \mathbf{H}(\mathbf{I} - \mathbf{R}_-)^{-1}\{\mathbf{I} + \mathbf{R}_+ \sum_{n=1}^{\theta} \mathbf{X}_-^{n-1}\}\mathbf{j}/\theta$$

with mean backlogs given, respectively, by:

$$\bar{N}_1 = \pi\{\mathbf{H}(\mathbf{I} - \mathbf{R}_-)^{-1}\mathbf{j} + \mathbf{HR} \sum_{n=1}^{\theta} \mathbf{X}_-^{n-1}\mathbf{j}\}/\bar{C}$$

$$= \pi\{\mathbf{H}(\mathbf{I} - \mathbf{R}_-)^{-1}\{\mathbf{I} + \mathbf{R}_+ \sum_{n=1}^{\theta} \mathbf{X}_-^{n-1}\}\mathbf{j}/\bar{C}$$

$$\bar{N}_2 = \pi\{\mathbf{H}(\mathbf{I} - \mathbf{R}_-)^{-1}\mathbf{j} + \mathbf{HR} \sum_{n=1}^{\theta} \mathbf{X}_-^{n-1}\mathbf{j}\}/\bar{C}bus$$

$$= \pi\{\mathbf{H}(\mathbf{I} - \mathbf{R}_-)^{-1}\{\mathbf{I} + \mathbf{R}_+ \sum_{n=1}^{\theta} \mathbf{X}_-^{n-1}\}\mathbf{j}/\bar{C}bus$$

NUMERICAL RESULTS

The scheme analysed above is tested in a system with $N = 19$ mobile users and a signal-to-noise ratio value of 3dB. All measures defined in the previous section are examined, and the most important results are reported. The steady-state probabilities are obtained by solving a linear system, using the direct approach of the Gaussian elimination method. The matrix $(\mathbf{I} - \mathbf{H}_-)^{-1}$ is found analytically, while the matrices $(\mathbf{I} - \mathbf{R}_-)^{-1}$ and $\mathbf{X}_-^{\theta-1}$ are calculated by using standard NAG routines.

In Figures 2 to 7 all the measures discussed in the previous section, with packet length $\theta = 5$ and attempt probability $\delta = 1.0$, are plotted against the arrival rate. Thus, in Figure 2 and (and Figure 3) the throughput S_1 (S_2) and the proportion of time spent in the contention period

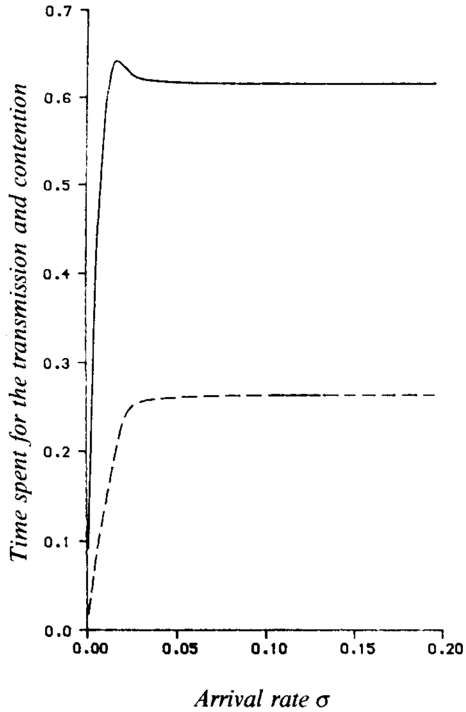

Figure 2. Arrival rate σ versus the proportion of the time spent in the transmission (throughput S_1: ——) and the contention (Ucon$_1$: - - -) period (N = 19, δ = 1.0, θ = 5, S/N = 3dB)

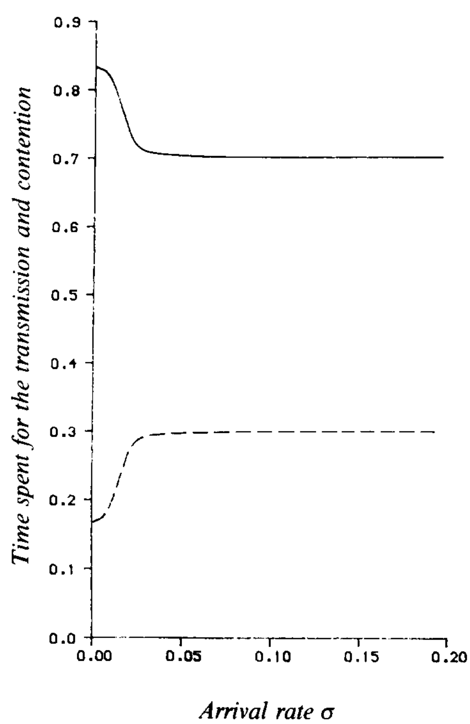

Figure 3. Arrival rate σ versus the proportion of the time spent in the transmission (throughput S_2: ——) and the contention (Ucon$_2$: - - -) period (N = 19, δ = 1.0, θ = 5, S/N = 3dB)

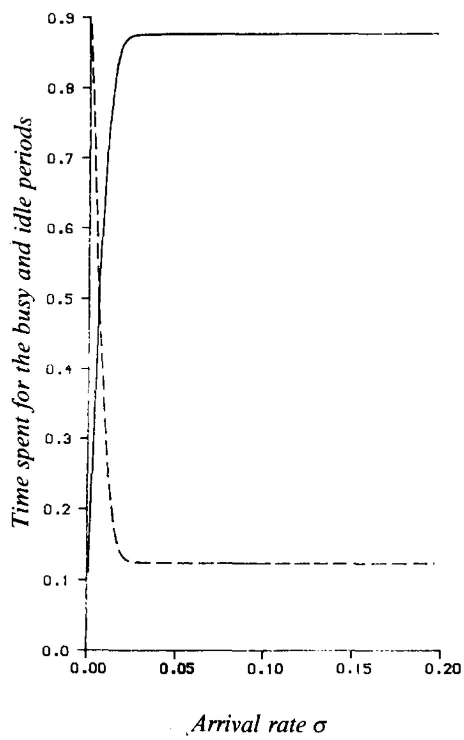

Figure 4. Arrival rate σ versus the proportion of the time spent in the busy (Ubus: ——) and the idle (Uidl: - - -) period (N = 19, δ = 1.0, θ = 5, S/N = 3dB)

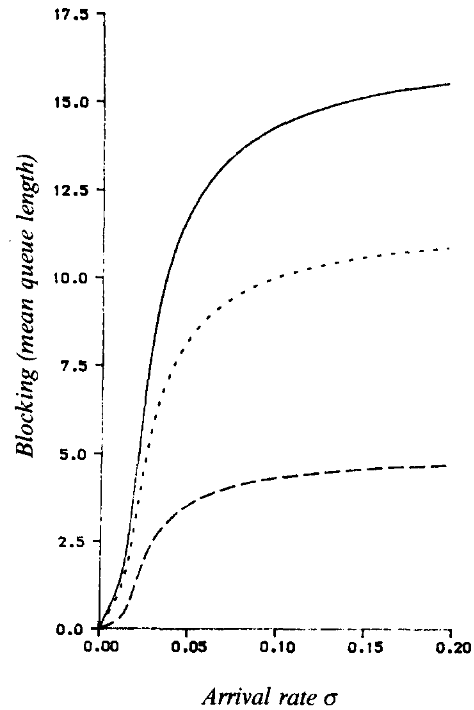

Figure 6. Arrival rate σ versus the mean queue length of the backlogged mobile users during the busy ($\overline{N}_2$: ——), the contention ($\overline{N}$con: - - -) and the transmission ($\overline{N}$tra: · · ·) period (N = 19, δ = 1.0, θ = 5, S/N = 3dB)

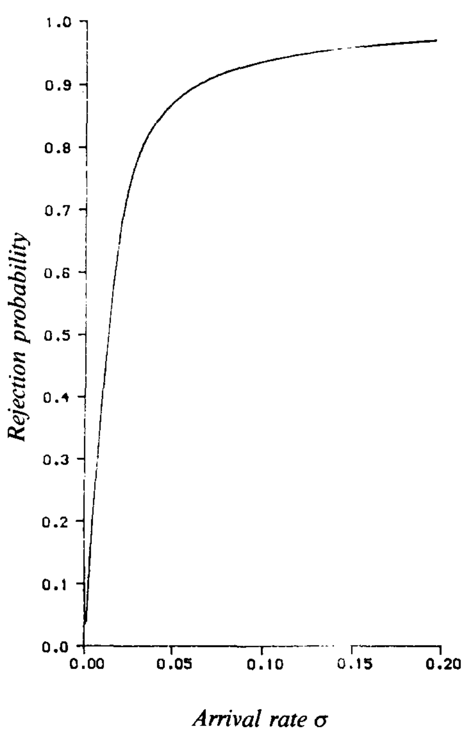

Figure 5. Arrival rate σ versus the regection probability (N = 19, δ = 1.0, θ = 5, S/N = 3dB)

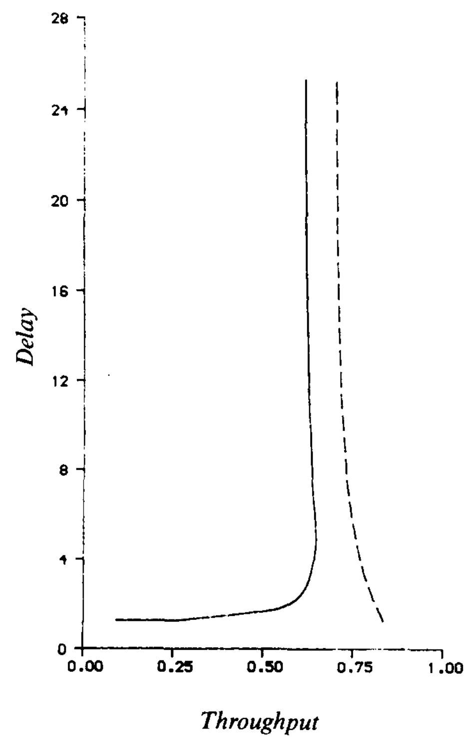

Figure 7. Throughputs S_1: —— and S_2: - - - versus the mean delay $\overline{D}$ (N = 19, δ = 1.0, θ = 5, S/N = 3dB)

$Ucon_1$ ($Ucon_2$) against the arrival rate are presented. Note that in both graphs all the measures involved are stable under overload. The values of S_2 and $Ucon_2$ are higher than those of S_1 and $Ucon_1$, respectively, due to the time spent in the idle period. Note that S_2 ($Ucon_2$) approaches its maximum (minimum) value as $\sigma \rightarrow 0$. This means that almost every incoming packet is transmitted successfully. As the traffic increases, S_2 decreases ($Ucon_2$ increases) at the expense of the other periods.

In Figure 4 the proportion of the time spent for the busy *Ubus* and idle *Uidl* periods against the arrival rate is presented. These measures seem to be both stable under overload. Note that for low traffic the idle period is longer than the busy period, but as the traffic increases the situation is reversed. In Figure 5 the rejection probability against the arrival rate is examined. It is expected that the rejection probability will increase with the packet length. This is due to the transmission period which is longer than the other two periods of the cycle, and therefore the packets have to wait more time in order to be served.

In Figure 6 the mean queue lengths of the backlogged mobile users (blocking) during the busy $\bar{N}_2$, contention $\bar{N}con$ and transmission $\bar{N}tra$ periods against the arrival rate are presented. As in the case with the rejection probability, it is expected that $\bar{N}_2$ will increase with the packet length. This is due to the increase in the blocking $\bar{N}tra$ in the transmission period. Figure 7 shows the throughput-delay characteristics. It appears that when the arrival rate is small, S_1 takes its lowest values while S_2 takes its maximum values. In this case the delay is small (almost every packet is successfully transmitted). However, as the arrival rate increases the situation is reversed. It is expected that as the packet length increases the throughputs S_1 and S_2 tend to obtain the same values.

In the sequel the throughput-delay performance for various values of the packet length θ and attempt probabilities δ is considered. The throughput S_1 *versus* the mean delay $\bar{D}$ is considered in Figures 8, 10, 12 and 14, while the throughput S_2 *versus* $\bar{D}$ is considered in Figures 9, 11, 13 and 15. Note that in Figures 8, 9, 10 and 11, five values of the packet length (θ = 1, 5, 15, 30, 50) are examined, keeping the attempt probability fixed at δ = 1.0 and 0.05, respectively. Similarly, in Figures 12, 13, 14 and 15, five values of the attempt probability (δ = 0.01, 0.05, 0.1, 0.2, 1.0) are examined, keeping the packet length fixed at θ = 5 and 30 slots, respectively.

It appears that as the users transmit longer (but fixed) packets, the throughputs S_1 and S_2 increase significantly at the expense of the idle and contention periods. Although the attempt probability δ plays an important role, especially when the traffic is light, it seems that it has no further effect under overload. In particular, if the packet length θ is long and as the attempt probability increases, the obtained throughput is almost the same.

Lastly, for small values of the attempt probability δ and as the traffic increases, packets seem to suffer to an excessive delay. The magnitude of this delay depends on the packet length. The smaller the packet length is the longer the corresponding delay, and *vice versa*. Although in a microcellular environment small packets are more

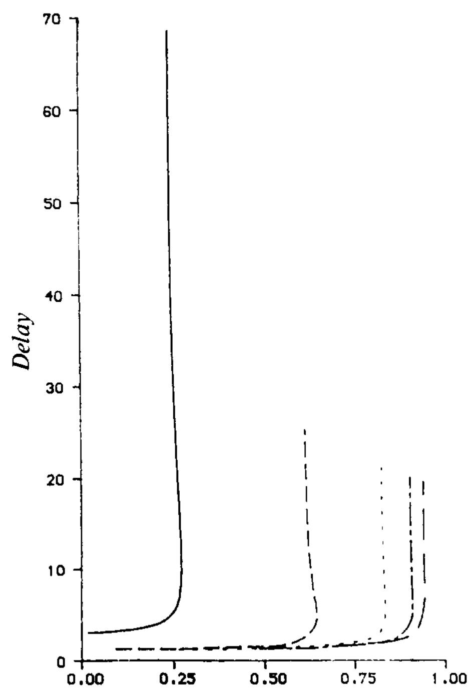

Figure 8. Throughput S_1 versus mean delay $\bar{D}$ for (a): θ = 1: ——; (b): θ = 5: - - -; (c): θ = 15: · · ·; (d): θ = 30: — · —; and (e): θ = 50: — —. (N = 19, δ = 1.0, θ = 5, S/N = 3dB)

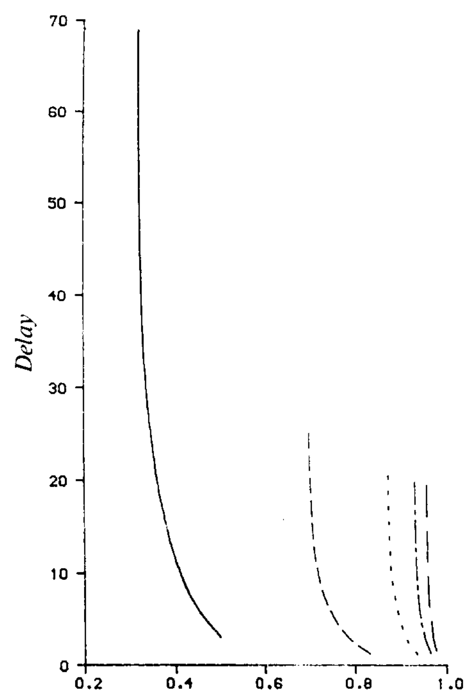

Figure 9. Throughput S_2 versus mean delay $\bar{D}$ for (a): θ = 1: ——; (b): θ = 5: - - -; (c): θ = 15: · · ·; (d): θ = 30: — · —; and (e): θ = 50: — —. (N = 19, δ = 1.0, θ = 5, S/N = 3dB)

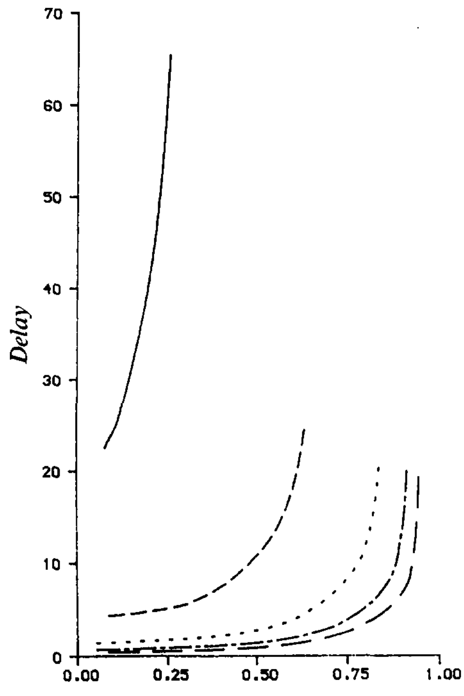

Figure 10. Throughput S_1 versus mean delay $\overline{D}$ for (a): $\theta = 1$: ——; (b): $\theta = 5$: - - -; (c): $\theta = 15$: · · ·; (d): $\theta = 30$: — · —; and (e): $\theta = 50$: — —. ($N = 19$, $\delta = 0.05$, $\theta = 5$, S/N = 3dB)

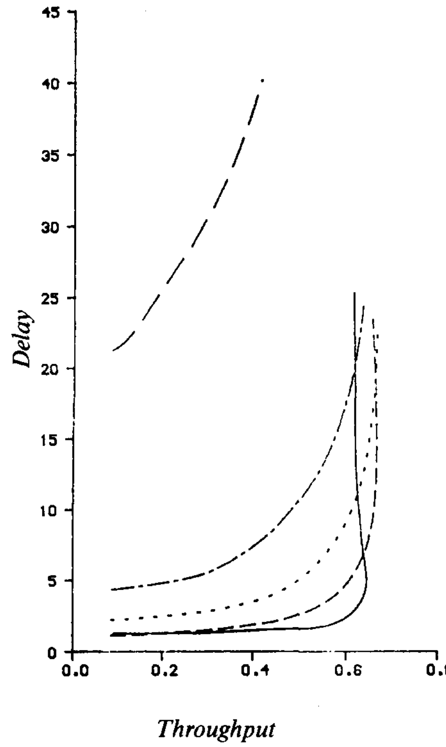

Figure 12. Throughput S_1 versus mean delay $\overline{D}$ for (a): $\delta = 1.00$: ——; (b): $\delta = 0.20$: - - -; (c): $\delta = 0.10$: · · ·; (d): $\delta = 0.05$: — · — ·; and (e): $\delta = 0.01$: — —. ($N = 19$, $\theta = 5$, S/N = 3dB)

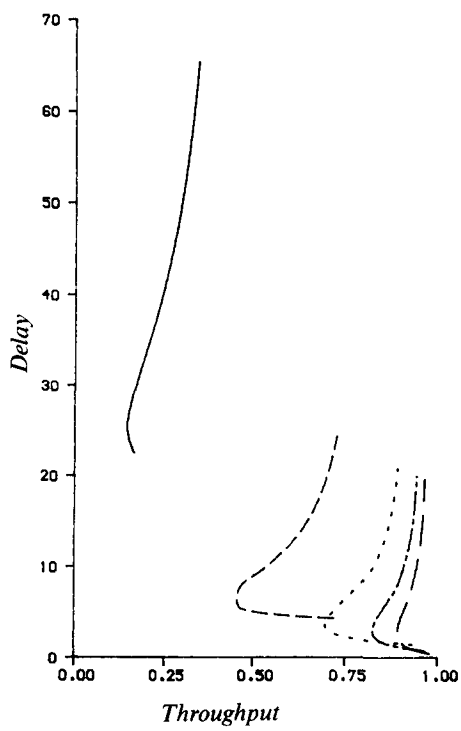

Figure 11. Throughput S_2 versus mean delay $\overline{D}$ for (a): $\theta = 1$: ——; (b): $\theta = 5$: - - -; (c): $\theta = 15$: · · ·; (d): $\theta = 30$: — · —; and (e): $\theta = 50$: — —. ($N = 19$, $\delta = 0.05$, $\theta = 5$, S/N = 3dB)

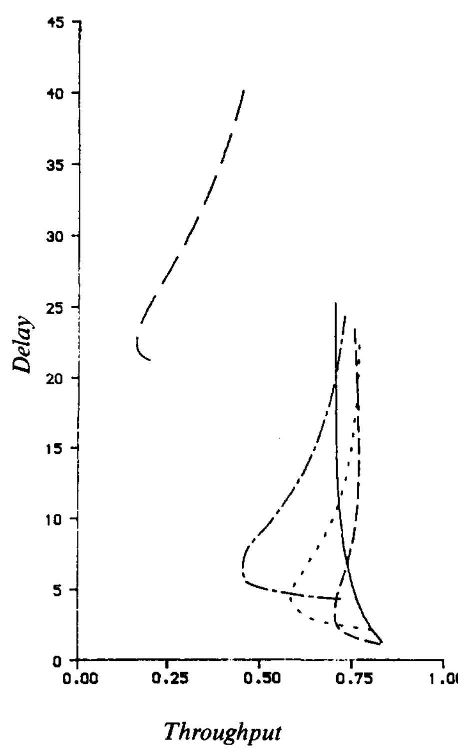

Figure 13. Throughput S_2 versus mean delay $\overline{D}$ for (a): $\delta = 1.00$: ——; (b): $\delta = 0.20$: - - -; (c): $\delta = 0.10$: · · ·; (d): $\delta = 0.05$: — · — ·; and (e): $\delta = 0.01$: — —. ($N = 19$, $\theta = 5$, S/N = 3dB)

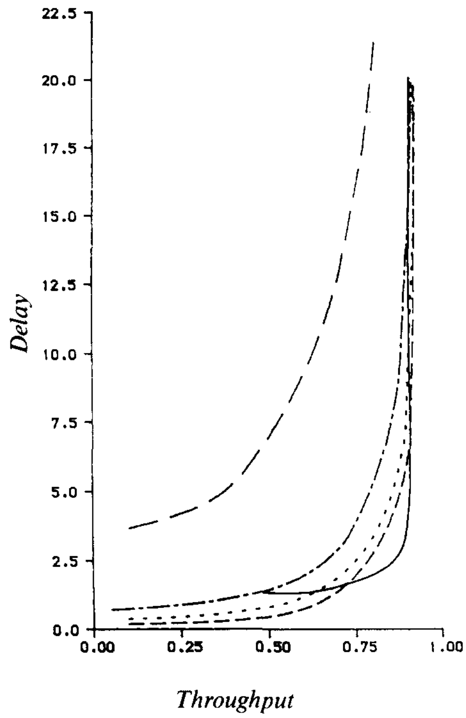

Figure 14. Throughput S_1 versus mean delay $\overline{D}$ for (a): $\delta = 1.00$: ——; (b): $\delta = 0.20$: - - -; (c): $\delta = 0.10$: · · ·; (d): $\delta = 0.05$: — · — ·; and (e): $\delta = 0.01$: — —. (N = 19, $\theta = 30$, S/N = 3dB)

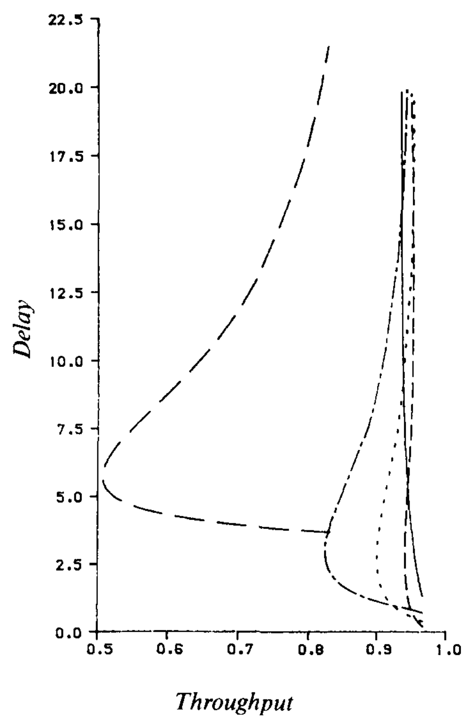

Figure 15. Throughput S_2 versus mean delay $\overline{D}$ for (a): $\delta = 1.00$: ——; (b): $\delta = 0.20$: - - -; (c): $\delta = 0.10$: · · ·; (d): $\delta = 0.05$: — · — ·; and (e): $\delta = 0.01$: — —. (N = 19, $\theta = 30$, S/N = 3dB)

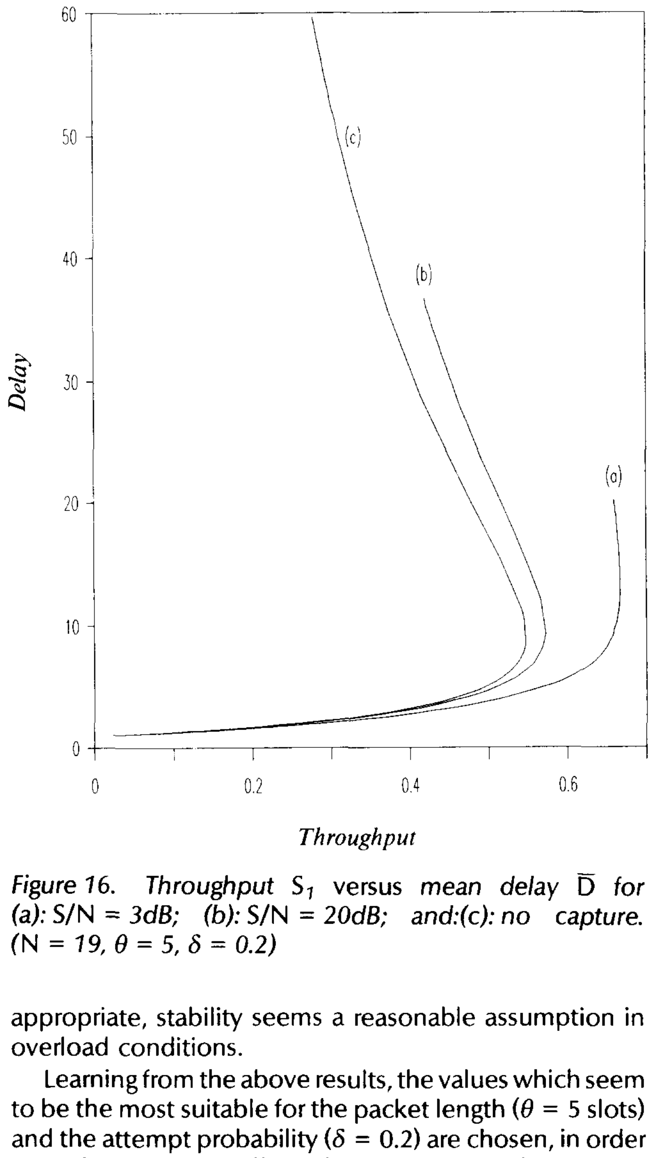

Figure 16. Throughput S_1 versus mean delay $\overline{D}$ for (a): S/N = 3dB; (b): S/N = 20dB; and:(c): no capture. (N = 19, $\theta = 5$, $\delta = 0.2$)

appropriate, stability seems a reasonable assumption in overload conditions.

Learning from the above results, the values which seem to be the most suitable for the packet length (θ = 5 slots) and the attempt probability (δ = 0.2) are chosen, in order to study the capture effect. Thus, in Figure 16, three values for signal-to-noise ratio are examined corresponding to 3dB, 20dB and no capture. The results are remarkable, in the sense that the throughput is substantially higher and the risk of instability is lower as the capture effect becomes more significant.

CONCLUSION

In this work the capture effect in a p-CSMA protocol suitable for a mobile radio telephone network, without hidden users, fading effects and propagation delay, was investigated. A Markov model was constructed by assuming that the packet collision does not necessarily destroy all the packets involved. It was shown that with a long packet length and a suitable reattempt probability, such a network increases its possibility of being stable, and it can throughput significantly more traffic than the corresponding p-persistent CSMA schemes without capture.

ACKNOWLEDGEMENTS

The authors would like to express their thanks to the anonymous referees, and to the Editor, for their valuable comments.

REFERENCES

1 **Abramson, N** 'The throughput of packet broadcasting channels' *IEEE Trans. Commun.* Vol 25 No 1 (January 1977) pp 117-128
2 **Metzner, J J** 'On improving utilization in Aloha networks' *IEEE Trans. Commun.* Vol 24 No 4 (April 1976) pp 447-448
3 **Lau, L and Leung, C** 'Performance of a power group divisions scheme for Aloha systems in a finite capture environment' *Electronics Lett.* Vol 24 No 15 (21st July 1988) pp 915-916
4 **Tobagi, F A and Kleinrock, L** 'Packet switching in radio channels: Part II - the terminal hidden problem in carrier sence multiple access and the busy tone solution' *IEEE Trans. Commun.* Vol 23 No 12 (December 1975) pp 1417-1433
5 **Namislo, C** 'Analysis of mobile radio slotted Aloha networks' *IEEE J. Selected Areas in Commun.* Vol 2 No 4 (July 1984) pp 583-588
6 **Tsiligirides, T** 'Analysis of a mobile radio-telephone packet switching network, with capture phenomena' *Proc. 4th UK Telecommunication Performance Eng. Workshop* University of Edinburgh, Scotland (19-21 September 1988) pp 87-95