International Journal of Computer Mathematics
Vol. 00, No. 0, Month 2004, pp. 1–10

NOTE ON THE FEEDBACK CONTROL ALGORITHMS USED IN THE HIGH-SPEED NETWORKS

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(Revised 21 January 2004; In final form 29 January 2004)

In this article, we analyze a linear feedback control algorithm particularly suited for the Available Bit Rate service class in an Asynchronous Transfer Mode (ATM) networks. We envisage the development of a closed-loop, fluid approximation model, in which the propagation delay is reflected across the network, while the rate of transmission and the queue occupancy are modeled as fluids. Using a fluid model has the advantage to permit a simplified study of the network behavior. The above model is described with the continuous-time system of delay-differential equations, which is solved semi-analytically. The contribution of this work is to provide a sending rate scheme, which is based on both a rate control function and a suitable fuzzy function for network load and delay. It is shown that the concept of fuzzy set theory can be proved beneficial in the analysis of network load and delay, whose uncertainty is an inherent characteristic. Finally, the developments in the area of time-delay systems control allow to compute exact stability bounds of the Round Trip Time (RTT) and thus to indicate if the connection is in a stable state.

Keywords: High-speed networks; Feedback control algorithms; Congestion control; Fuzzy sets

ACM Classification: C.2.1; C.2.2

1 INTRODUCTION

In a Broadband Integrated Service Data Network, congestion is defined as a condition of an Asynchronous Transfer Mode (ATM) network, where the network does not meet a stated performance objective. By contrast, a traffic control, such as the Connection Admission Control (CAC), defines a set of actions taken by the network to avoid congestion. Because of the uncertainty in the traffic flows of multimedia services, network congestion may still occur even though an appropriate CAC scheme is provided. To prevent the Quality of Service from severely degrading during short-term congestion, an appropriate control scheme is required.

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ISSN 0020-7160 print; ISSN 1029-0265 online
DOI: 10.1080/00207160410001684316

In such cases, one of the basic problems arising is the presence of propagation delays, which pose a challenge for stability.

Since the ATM Forum decided to use a closed-loop rate based congestion control scheme as the standard for the Available Bit Rate (ABR) service, several feedback control schemes were proposed in the literature [1, 2, 3]. In most of the proposed algorithms, control decisions are based on the deviation of the state of the system from the target value, or of the sign of such deviation. Analysis and numerical simulations reported that the stability of such congestion control algorithms in the presence of propagation delays resulted in an undesirable, bounded, oscillatory behavior. However, the key obstacle in order to eliminate such oscillations is the single control parameter assumed by a large number of such algorithms, leading to an unstable equilibrium point.

For the shake of simplicity, this research is based on a simple continuous-time deterministic model derived from Ref. [1], modeling a single connection between a traffic source and a network node (Fig. 1).

Let

- $\lambda(t)$ be the source rate at time t, controlled by access regulator.
- μ be the constant transmission capacity of a distant node.
- $Q(t)$ be the current buffer size at time t.
- q_0 be the optimal queue size (threshold of the buffer with no congestion).
- τ_1 and τ_2 be the propagation delays from the source to the bottleneck and back to the source, respectively.
- RTT be the round-trip delay ($\tau = \tau_1 + \tau_2$).

The control objective is to adapt $\lambda(t)$ to μ dynamically, while keeping $Q(t)$ at an acceptable level. The algorithm takes into consideration the deviations of $Q(t)$ around the threshold between two consecutive control time intervals. We assume that the control interval, say r, presents an additional delay, which is generally not equal to the round-trip delay τ. Since the deviations are weighted using two linear gain parameters, say A and B, the dynamics of the switching node in the network may be described with the following system of delay-differential equations:

$$Q'(t) = \lambda(t - \tau_1) - \mu, \tag{1}$$

$$\lambda'(t) = -A[Q(t - \tau_2) - q_0] - B[Q(t - \tau_2 - r) - q_0]. \tag{2}$$

Differentiating the first equation above with respect to t and setting $z(t) = Q(t) - q_0$, we obtain the following second-order delay-differential equation:

$$z''(t) + \alpha z(t - \tau) + \beta z(t - \tau - r) = 0 \tag{3}$$

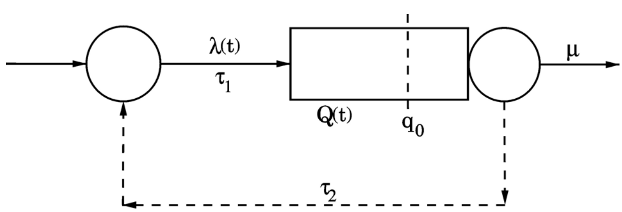

FIGURE 1 The basic model with a single VC traversing an ABR node with propagation delay $\tau = \tau_1 + \tau_2$.

or in the normalized time scale $T = t/\tau$:

$$z''(T) + \alpha z(T-1) + \beta z(T-1-\delta) = 0, \tag{4}$$

where $\alpha = A\tau^2$, $\beta = B\tau^2$, and $\delta = r/T$.
The characteristic equation of the above delay differential equation is given by:

$$h(s) = s^2 \mathrm{e}^{(\delta+1)s} + \alpha \mathrm{e}^{\delta s} + \beta = 0 \tag{5}$$

The above equation has an infinite number of roots θ_i and their location on the complex plane determines the asymptotic behavior of $z(T)$ around the equilibrium point 0. The degree of stability $\theta = \sup_i\{\mathcal{R}\theta_i\}$ guarantees the asymptotic convergence of $z(T)$ with the exponential rate θ: $|z(T)| \leq \zeta \mathrm{e}^{-\theta T}$, for some value of ζ. Since the principal root is negative and real, the choice of damping parameters of the coefficients α and β guarantees the exponential and non-oscillatory convergence of the algorithm [1]. Note that the free of delays ($r = \tau = 0$) system of Eq. (3) oscillates, *i.e.* it is unstable, since the corresponding characteristic equation has its roots on the imaginary axis. As seen in Ref. [4], the use of a small delay r in the model with $\tau = 0$ will induce the stability of the corresponding continuous model.

Recently, some interesting results have been provided [5, 6] in the analysis of the stability problem of Eq. (3) in the delay-parameter space $O\tau r$. They derived some sets of intervals of the form $[\underline{r}_i, \overline{r}_i]$ and $[\underline{\tau}_i, \overline{\tau}_i]$, which are function of the parameters (α, β) such that the system of Eqs. (1) and (2) is asymptotically stable. Note that parameters α, β, and r determine the exact bounds of stability for τ and therefore the variable τ can be used to characterize the stability of the network. Depending on the parameters that control the source, the measured RTT can indicate if the connection is in stable state or not by checking if the measured RTT belongs to some pre-computed intervals $[\underline{\tau}_i, \overline{\tau}_i]$.

For this analysis, both delays (τ, r) are considered as free parameters of the system, whereas the gains (α, β) are fixed. Parameters α, β, and r can be used to control the sending rate $\lambda(t)$ of the source, due to a rate control function $\lambda(t) = f(\alpha, \beta, r)$ that can be derived either from Eq. (3), or differently. Although such a control function has not been considered yet, one may induce that to increase the sending rate, the parameter α must be decreased, and the parameter $|\beta|$ must be increased. Inversely, to decrease the sending rate, α must be increased and $|\beta|$ must be decreased.

The above analysis is useful in a sense that it provides explicitly, but rather complicated in mathematical terms upper and lower stability/instability bounds that depend on additional delay and RTT. However, the problem of choosing the appropriate function $\lambda(t) = f(\alpha, \beta, r)$ is open. Therefore, the objective of this study is to determine a suitable function for the sending rate $\lambda(t)$, and to compute the exact stability of the RTT that indicates if the connection is in a stable state or not. Focusing on this direction, the article addresses both the rate control scheme described above and a reasoning issue, by means of introducing a suitable fuzzy function for network load and delay. Currently used methods for analysis of network information, such as load and delay data, are inadequate, because they do not tolerate uncertainty.

Uncertainty refers to the imperfect and not exact knowledge concerning some domain of interest. The uncertainty is an inherent feature of network traffic data and may arise through incomplete information associated to them, or because of the presence of varying concentration of attributes, and finally, because of the use of the use of qualitative descriptions of their attribute values and relationships. This work shows that the concept of fuzzy set theory can be proved beneficial in the analysis of network load and delay, whose uncertainty is an inherent characteristic. This is largely due to the underlying membership concept of the classical set theory, according to which a set has precisely defined boundaries and an element has either full or no membership in a given set (Boolean logic). Nowadays, many control engineering researchers are looking at the consequences of fuzzifying set theory. Fuzzy logic and fuzzy relations are the most important in order to understand how fuzzy rules work and how they are related with the network traffic data.

The article is organized as follows. The next section provides the framework in the fuzzy analysis of network data. First, a representation issue: it is shown how uncertainty, which characterizes network traffic features, may be incorporated into the fluid model; and second, a reasoning issue: it is shown how fuzzy logic methodologies may be incorporated into the basic data interpretation operations. Section 3 provides the analysis and definition of the suitable control function so that to be able to compute the sending rate. It also presents the congestion detection algorithm. Finally, Section 4 concludes the discussion by summarizing the contributions of the article and giving hints for future research.

2 FUZZINESS AND CONGESTION

The mathematical foundations of fuzzy logic rest on *fuzzy set theory*, which can be thought of as a generalization of classical set theory. Fuzzy sets are a further development of the mathematical concept of a set. A conventional *set* is any collection of distinct elements (objects, numbers, symbols, etc.), which can be treated as a whole. For this reason we also call them *crisp sets*. The terms *set, collection*, and *class* are synonyms, just as the terms *item, element*, and *member*. A set can be specified by its members, they characterize a set completely. The list of members specifies a *finite set*. There is no way to list all elements of an *infinite set*, it must instead state some property, for example a predicate, which characterizes the elements in the set. That set is defined by the elements of the universe of disclosure, which make the predicate true. So there are two ways to describe a set; explicitly in a list or implicitly with a predicate.

Many sets have more than an *either–or* criterion for membership. Given a universe of disclosure $Z = \{z|z \text{ is an object}\}$, a fuzzy subset A in Z is defined such that for any $z \in Z$ there is a *membership function* μ $(0 \leq \mu \leq 1)$, which specifies the degree to which z belongs to fuzzy subset A. The membership function is denoted by $\mu_A(z)$ and can take any value between zero and one, where zero indicates no membership and one indicates full membership. Since $\mu_A(z)$ of z in A specifies the extent to which z can be regarded as belonging to the set A, it is also known as the *degree of membership (d.o.m.) of z in A*. Specifically, $\mu_A(z) = 1$, if $z_1 \leq z \leq z_2$; and $\mu_A(z) = 0$, if $z < z_1$ or $z > z_2$, where z_1 and z_2 define the exact boundaries of set A. For instance, if the boundaries between *lightly loaded, medium loaded*, and *heavy loaded* were to be set at $z_1 = 20\%$ and $z_2 = 80\%$ of the network load, then the membership function defines all *medium loads*. Notice that classical sets allow only binary membership functions (*i.e.* true or false).

In the above paradigm, just like an algebraic variable takes numbers as values, a *linguistic variable* takes words or sentences as values. The set of values that it can take is called *term set*. Each value in the term set is a *fuzzy variable* defined over a *base variable*. The base variable defines the universe of discourse for all the fuzzy variables in the term set. Thus, the hierarchy is linguistic variable → fuzzy variable → base variable.

Example 1 Let a linguistic variable with a label *Load* take values from the state set L, say. Terms (values) of this linguistic variable, which are fuzzy sets, could be *VeryLight, Light, MoreOrLessLight, NotSoLight, Medium, AlmostHeavy, Heavy, VeryHeavy*. Each term is a fuzzy variable defined on the base variable, which might be the scale from 0% to 100% of the network load.

A *primary term* is a term (or a set) that must be defined *a priori*. In the example just considered, the primary terms are *Light, Medium*, and *Heavy*, whereas the rest are modified terms (or sets). Generally, the values of a linguistic variable may be *compound*, namely, values constructed through the use of *primary values* and linguistic modifiers such as *Not, Very, Rather, Almost*, and *MoreOrLess*. The above three primary terms, may be modeled as shown in Figures 2 and 3. Thus in Figure 2, we have three distinct fuzzy values, while in Figure 3 we have three (piece-wise) continuous membership functions $\mu_L^{Light}(z)$, $\mu_L^{Medium}(z)$, $\mu_L^{Heavy}(z)$ modeling the words *Light, Medium*, and *Heavy*, respectively. Any crisp value of the network load (*e.g.* 60%) has a unique *d.o.m.* to each fuzzy value of *Load*. In Figure 3, for example, crisp load 60% is *Light* to a degree zero, *Medium* to a degree 0.65, and *High* to a degree 0.35. Notice that a fuzzy membership function is different from a statistical probability distribution. This may be illustrated in the following example.

Example 2 Consider the statement: The network is $w\%$ loaded, where:

$$w \in \Omega = [\,(0^{+}\text{–}5] \quad (5\text{–}20] \quad (20\text{–}35] \quad (35\text{–}50] \quad (50\text{–}65] \quad (65\text{–}80] \quad (80\text{–}95] \quad (95\text{–}100]].$$

Then we may associate a probability distribution p by observing the network load for a large number of time slots.

$$\Omega = [\,(0^{+}\text{–}5] \quad (5\text{–}20] \quad (20\text{–}35] \quad (35\text{–}50] \quad (50\text{–}65] \quad (65\text{–}80] \quad (80\text{–}95] \quad (95\text{–}100]\,]$$
$$p = [0 \quad 0.2 \quad 0.6 \quad 0.2 \quad 0 \quad 0 \quad 0 \quad 0\,]$$

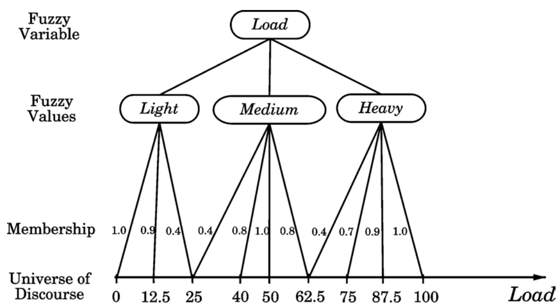

FIGURE 2 The linguistic variable *load* and a set of discrete fuzzy values.

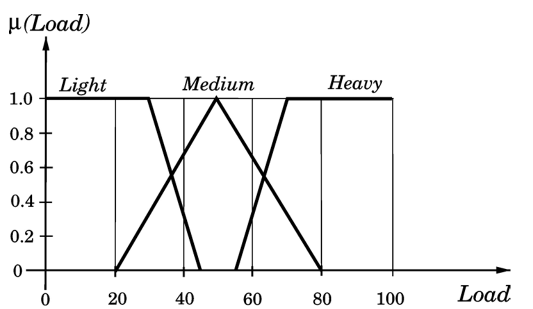

FIGURE 3 Membership functions $\mu(S)$ used for describing the primary values *Light, Medium*, and *Heavy* of the linguistic variable *Load*.

A fuzzy set expressing the grade of load of a network could be given by the following *possibility distribution*:

$$\Omega = [\,(0^{+}\text{–}5] \quad (5\text{–}20] \quad (20\text{–}35] \quad (35\text{–}50] \quad (50\text{–}65] \quad (65\text{–}80] \quad (80\text{–}95] \quad (95\text{–}100]\,]$$

$$\pi = [\,1 \quad 1 \quad 1 \quad 1 \quad 0.8 \quad 0.6 \quad 0.4 \quad 0.2\,]$$

The example shows that the possibility for $w \in (35\text{–}50]$ (in %) is 1, whereas the probability is only 0.2. This means that a possible event does not imply that it is probable. However, if it is probable it must also be possible. You might view a fuzzy membership function as the distribution of a network administrator, in contrast with the statistical distribution based on observations.

Fuzzy set theoretic *operations* provide the counterpart operations to those of classical set theory [7, 8]. In other words, logical operations with fuzzy sets are generalizations of usual Boolean algebra applied to observations that have partial membership of more than one set. The standard operations of *union, intersection*, and *complement* of fuzzy sets A and B, defined over some domain Ω, create a new fuzzy set whose membership function is defined through the min $\wedge$ and max $\vee$ operations, namely:

$$\mu_{A\cap B}(z) = \mu_A(z) \wedge \mu_B(z) = \min\{\mu_A(z), \mu_B(z)\}, \quad \forall z \in \Omega, \tag{6}$$

$$\mu_{A\cup B}(z) = \mu_A(z) \vee \mu_B(z) = \max\{\mu_A(z), \mu_B(z)\}, \quad \forall z \in \Omega, \tag{7}$$

$$\mu_{\bar{A}}(z) = 1 - \mu_A(z), \quad \forall z \in \Omega. \tag{8}$$

Consider the classification of states based on the set L of load (%) with lexical values *Light, Medium, Heavy*, and a second classification based on the set D of delay (%), with lexical values *Small, Moderate, Large*, respectively. In the above, and without loss of generality, we assume that *d.o.m.* for *Heavy* load is $\mu_L^{Heavy}(s) = 0.9\%$ or 90%, and similarly the *d.o.m.* for the *Small* delay is $\mu_D^{Small}(s) = 0.15$ or 15%. For each state s, the *d.o.m.* value that provides an overall measure regarding the load and the delay is derived by the intersection operation, namely, by taking:

$$\mu_{L\cap D}(s) = \mu_L^{Heavy}(s) \wedge \mu_D^{Small}(s) = \min\{\mu_L^{Heavy}(s), \mu_D^{Small}(s)\} = 0.15.$$

Similarly, the *d.o.m.* value that provides an overall measure regarding the load or the delay is derived by the union operation, namely, by taking:

$$\mu_{L\cup D}(s) = \mu_L^{Heavy}(s) \vee \mu_D^{Small}(s) = \max\{\mu_L^{Heavy}(s), \mu_D^{Small}(s)\} = 0.9.$$

Finally, the non-loaded network is derived by:

$$\mu_{\bar{L}}(s) = 1 - \mu_L(s) = 1 - \mu_L^{Heavy}(s) = 1 - 0.9 = 0.1.$$

A problem that arises in this case is that only one of the participating *d.o.m.* values dominates by assigning its value to the whole decision criterion. In this way, the contribution of the other *d.o.m.* values is eliminated. Several other functions have been proposed for logical operations on observations that have partial membership on more than one set. These functions are more complex to compute and interpret; however, they provide more expressive and accurate results.

One of those is the *energy metric* [9], in which k fuzzy sets ($F_1, F_2, \ldots, F_k$) defined over some domain Ω create a new fuzzy set F, with membership function given by the following equation:

$$\mu_F(z) = \sum_{i=1}^{k} [\mu_{F_i}(z)]^b, \tag{9}$$

where $b \in Z^+$. By applying this equation for $b = 2$ (quadratic measure), the large weight values *d.o.m.* are amplified, while the small values are nearly eliminated. Assuming the previous example, the overall measure characterizing each state, regarding the level of load and delay, the energy function may be provided by the following equation:

$$\mu_{L,D}^{Heavy,Small}(s) = [\mu_L^{Heavy}(s)]^2 + [\mu_D^{Small}(s)]^2. \tag{10}$$

Equation (9) is more flexible and expressive than Eq. (7) because its overall measure is explicitly affected by the *d.o.m.* values of each state on both fuzzy sets A and B. Specifically, assume two states s_1 and s_2 with *d.o.m.* values: s_1: $\{\mu_L^{Heavy}(s_1) = 0.8$ and $\mu_D^{Small}(s_1) = 0.4\}$ and s_2: $\{\mu_L^{Heavy}(s_2) = 0.6$ and $\mu_D^{Small}(s_2) = 0.4\}$. The overall measure provided by Eq. (6) is 0.4 for both states, while that derived by Eq. (10) is 0.80 for s_1 and 0.52 for s_2. Clearly, the energy metric provides an ordering of the two states. It says that s_1 satisfies better the two criteria posed by the network administrator (*i.e.* level of load and delay). This feature is very beneficial for decision criteria, which combine multiple sets and lexical values, while ordering of the qualified states is required (*i.e.* find the five most load and delay states of a network during a certain period).

3 CONTROL FUNCTION

3.1 Application to Congestion

The choice of membership function, by means of its shape and form, is crucial. Generally, there are many possible approaches of deriving membership functions for fuzzy sets, however, the following two are the most interesting. The first approach is based on data driven multi-variate procedure and is called Similarity Relation Model. It is analogous to that taken by cluster analysis and numerical taxonomy, in that the value of the membership function is a function of the classifier used. The second approach is simpler and is used more frequently. It is based on an imposed expert model, which called Semantic Import Model. The membership grade is assigned using a membership function, derived by an objective or subjective process depending on the way in which the experts agree to define classes. This model is useful in situation where users have a very good idea of how to group data, but for various reasons they are constrained from using the standard Boolean model.

Several functions exist and the most suitable, by means of defining flexible membership grades, which can be easily adapted to specific requirements, are used. For example, to model the primary values *Light, Medium*, and *Heavy* of the fuzzy variable *Load* shown in Figures 2 and 3, and similarly, the primary values *Small, Moderate*, and *Large* of the fuzzy variable *Delay* a generalized bell function of the form:

$$\mu_A(z) = \left[1 + \left(\frac{z - \upsilon}{\rho}\right)^{2\xi}\right]^{-1} \tag{11}$$

may be used. The adaptable parameters ρ, ξ, and υ are used to adjust the overall form of $\mu_A(z)$. Parameter ρ adjusts the width of the membership function, ξ determines the extent of fuzziness, and υ is the value of z at the central concept, namely, it describes the location of the peak of the membership function. Usually, the fussy sets are normalized. Therefore, there exist at least one point of the universe of discourse where the membership function reaches unity, whereas in fuzzy linguistic description this requirement is relaxed. The fuzzy values determined above ought to be convex, but not necessarily normal.

As it has been stated in the introduction, our interest is to define a control rate function $f(\alpha, \beta, r)$ that can be used to regulate the sending rate $\lambda(t)$ of the source. The variables α, β, and r are parameters while τ is the RTT measured with acks representing the implicit feedback about the network. Notice that setting $\lambda(t) = f(\alpha, \beta, r)$, the rate control function f can be Q1 derived from Eqs. (1) and (2). This approach requires to solve a system of DDEs, possibly through the use of some iterative numerical method, without the need to provide an analytical form for the function f. A similar procedure has been followed in Ref. [10]. On the basis of the work proposed in Ref. [5] (the main results are shown in Appendix), it may be possible to define sets of intervals for τ and r that ensure the stability of system of Eqs. (1) and (2).

3.2 Congestion Detection Algorithm

The advantage of the above analysis is to provide explicitly upper and lower stability/instability bounds in terms of additional delay and round-trip times. In the following, we provide an algorithm to be used to detect congestion. It is based on the design ideas suggested in Ref. [5]; however, its main difference lies in the introduction of the membership function $\mu_{L,D}$ as an alternative choice of the control rate function $f(\alpha, \beta, r)$, which regulates the source sending rate $\lambda(t)$.

The algorithm is divided into three faces. In the first phase (initialization), it determines the intervals $[\underline{\tau}_i, \overline{\tau}_i]$ using Theorem 1 (Appendix A) and sets τ equal to the minimum of RTT. In addition, in this stage, the algorithm calculates the membership functions μ_L, μ_D for network load and delay, as they defined in Eq. (11), and uses them to determine the quadratic energy metric $\mu_{L,D}$, already defined in Eq. (10). In the second phase (receiving an ack), the algorithm measures the RTT. Then it checks if the τ (measured RTT) belongs to some of these pre-calculated intervals. Depending on the parameters that control the source the measured RTT can indicate if the connection is in a stable state. On the basis of this result, the algorithm adjusts the parameters α and β, and modifies the control function $f(\alpha, \beta, r)$. Thus, the algorithm is able to determine the sending rate $\lambda(t)$ as an appropriate choice between the control function f and the quadratic energy measure $\mu_{L,D}$. The algorithm lasts with the third phase (sending a packet), in which it calculates the inter-packet gap delay.

The algorithm:

- **Initialization:**
  - set $\alpha = \alpha_0$, $\beta = \beta_0$, and $r = r_0$. The values α_0, β_0, and r_0 have been chosen in order to obtain a very large sending rate.
  - Compute the intervals $[\underline{\tau}_i, \overline{\tau}_i]$ using equation (Eq. (A2)).
  - set $\tau = \min RTT$.
  - Compute the membership functions μ_L, μ_D, and $\mu_{L,D}$ using Eqs. (10) and (11).

- **Receiving an ack:**
  - set τ = measured RTT.
  - **if** $\tau \notin [\underline{\tau}_i, \overline{\tau}_i]$ **then**

∗ Network is unstable. Sending rate has to be decreased. Increase α, decrease $|\beta|$.
∗ Choose r in order to optimize the sending rate, the RTT bounds, or the robustness of the end-to-end connection against further small variations of traffic (Eq. (A1)).
∗ Re-compute the intervals $[\underline{\tau}_i, \overline{\tau}_i]$ using equation (Eq. (A2)).
∗ Compute the new sending rate $\lambda(t) = \min\{f(\alpha, \beta, r), \mu_{L,D}\}$.

else

∗ Network is stable. Sending rate can be increased. Decrease α, increase $|\beta|$.
∗ Choose r in order to optimize the sending rate, the RTT bounds, or the robustness of the end-to-end connection against further small variations of traffic (Eq. (A1)).
∗ Re-compute the intervals $[\underline{\tau}_i, \overline{\tau}_i]$ using equation (Eq. (A2)).
∗ Compute the new sending rate $\lambda(t) = \max\{f(\alpha, \beta, r), \mu_{L,D}\}$.

end if

- **Sending a packet:**
  - Compute the Inter-Packet Gap delay; IPG = packet size/$\lambda(t)$
  - Send the packet respecting IPG.

4 CONCLUSIONS

This work deals with applications to best effort traffic of some fluid approximations models encountered in the literature, and to their analysis in terms of RTT. The developments in the area of time-delay systems control allow to compute optimal bounds guaranteeing stability of the corresponding continuous-time schemes, which define some degree of robustness in terms of delays. Depending on the parameters that control the source the measured RTT can indicate if the connection is in a stable state. On the basis of this result, the algorithm adjusts the parameters α, β, and modifies the control function $f(\alpha, \beta, r)$. The contribution of this work is to provide a sending rate scheme, which takes into account the rate control function and a suitable fuzzy function for network load and delay. It is shown how the useful concepts of fuzzy set theory may be adopted for the analysis of network data, such as load and delay, whose uncertainty is an inherent characteristic. The purpose is to take advantage on the robustness issues of the fluid approximation in order to improve the behavior of the discrete-time network. Future research includes the simulation of the presented prototype [11] for the application domain of congestion control with fuzzy set methodologies incorporated in both the analysis and experimental study on the choice of membership functions for the set of constrains posed by the network administrators.

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A APPENDIX

This appendix provides the main results of the work proposed in Ref. [5]. It defines sets of intervals for τ and r that ensure the stability of system of equations (1) and (2).

THEOREM 1 (Stability) *The system of equations* (1) *and* (2) *with* $b < 0$ *and* $a > |b|$ *is stable for all delays* r *and* τ *satisfying the following constrains*:

$$\underline{r}_i(\alpha, \beta) = \frac{2i\pi}{\sqrt{\alpha - |\beta|}} < r < \frac{(2i+1)\pi}{\sqrt{\alpha + |\beta|}} = \overline{r}_i(\alpha, \beta), \quad i = 0, 1, 2, \ldots, \tag{A1}$$

$$\underline{\tau} = 0 < \tau(\alpha, \beta, r) = \overline{\tau}(\alpha, \beta, r) = \min_{\omega_s} \frac{1}{\omega_s} \arctan\left[\frac{-b\sin(\omega_s r)}{\alpha + \beta\cos(\omega_s r)}\right], \tag{A2}$$

where ω_s *belongs to the set of positive solutions of the equation*: $\omega^4 + \omega^2 + 2\alpha\beta\cos(\omega r)$.

This theorem defines sets of intervals for τ and r that ensure the stability of system of equations (1) and (2). Other values of these parameters lead to instability. The range of i guaranteeing the inequalities in Eqs. (12) and (13) can be computed as follows:

$$i \in N \quad \text{and} \quad 0 \leq i < \frac{1}{2}\frac{1}{\sqrt{(\alpha + |\beta|)/(\alpha - |\beta|)} - 1} \tag{A3}$$

THEOREM 2 (Instability due to τ) *Assume that the delay* τ *of Eq.* (A1) *and the value* $\hat{\omega}_s$ *of Eq.* (13) *satisfy the inequality:* $\hat{\omega}_s^3 > (1/2)\alpha|\beta|r\ \sin(\hat{\omega}_s r)$. *Then we have*:

1. *If* $\tau = \overline{\tau}(\alpha, \beta, r) + \epsilon$ *with* $\epsilon > 0$ *sufficiently small, the system of equations* (1) *and* (2) *is unstable.*
2. *If Eq.* (A2) *has only one positive solution, then there does not exist any positive value of* $\tau > \underline{\tau}(\alpha, \beta, r)$ *such that system of equations* (1) *and* (2) *is asymptotically stable.*

THEOREM 3 (Instability due to r) *If for a pair* (α, β) *with* $\beta < 0$ *and* $\alpha > |\beta|$, *the delay* r *does not satisfy the constrains of Eq.* (A1), *then the closed-loop system is unstable for any delay* τ *satisfying Eq.* (A2) *and the conditions of the Theorem* 2.

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Journal … **Internaltion Journal of Computer Mathematics** Article ID …**GCOM 041029**

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